Using a signature-based machine learning model to analyse a psychiatric stream of data

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(JOINT WORK WITH T. LYONS AND K. SAUNDERS)

Mathematical Institute
University of Oxford

7th Berlin-Oxford Meeting

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Oxford Mathematics

Signature of a path

Continuous paths with finite p-variation

- Given $p \geq 1$ and $X \in \mathcal{C}([s,t],\mathbb{R}^d)$ with $s < t$ we define

$$\|X\|_{p,[s,t]} := \sup_{\{t_i\}_i \subset [s,t]} \left(\sum_i \|X_{t_i} - X_{t_{i-1}}\|^p \right)^{1/p} .$$

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- $\mathcal{V}^p([s,t],\mathbb{R}^d) := \{X \in \mathcal{C}([s,t],\mathbb{R}^d) : \|X\|_{p,[s,t]} < \infty\}.$

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Definition (Signature of a continuous path)

Let $X \in \mathcal{V}^p([0,T],\mathbb{R}^d)$ so that the following integration makes sense. The signature of X is defined as

$$S(X) = (1, X^1, X^2, \dots) \in \bigoplus_{n=0}^{\infty} (\mathbb{R}^d)^{\otimes n}$$

where

$$X^n = \int \dots \int_{0<u_1<u_2<\dots<u_n<T} dX_{u_1} \otimes \dots \otimes dX_{u_n} \quad \forall n \geq 1.$$

Definition (Truncated signature of a continuous path)

Similarly, we define, for $n \geq 0$,

$$S^n(X) := (1, X^1, X^2, \ldots, X^n).$$

- We have two data sets: a known set of known input-output pairs (the *training set*), $\{X_i, Y_i\}_i$, which is used to train the model, and a set of inputs that is used for testing (the *out-of-sample set*).

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- Features play an important role in machine learning.

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Theorem (B. Hambly, T. Lyons)

The signature of a path with bounded variation is unique up to tree-like equivalence.

Theorem

Let $X \in \mathcal{V}^1([0,T], \mathbb{R}^d)$ be a path with bounded variation. Then, given $1 \leq i_1, i_2, \ldots, i_n \leq d$ we have

$$\left\| \int \cdots \int_{0<u_1<u_2<\ldots<u_n<T} dX^{i_1}_{u_1} \ldots dX^{i_n}_{u_n} \right\| \leq \frac{\|X\|^n_{1,[0,T]}}{n!}.$$

- Given a training set $\{R_i, Y_i\}_{i=0}^{N}$, of input-output pairs, where $R_i = \{t_{ij}, r_{ij}\}_j$ is a stream of data, construct a new set $\{X_i, Y_i\}_{i=0}^{N}$ with $X_i \in \mathcal{V}^1$.

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- Compute $\{S^n(X_i), Y_i\}_{i=0}^{N}$ for some $n \in \mathbb{N}$.

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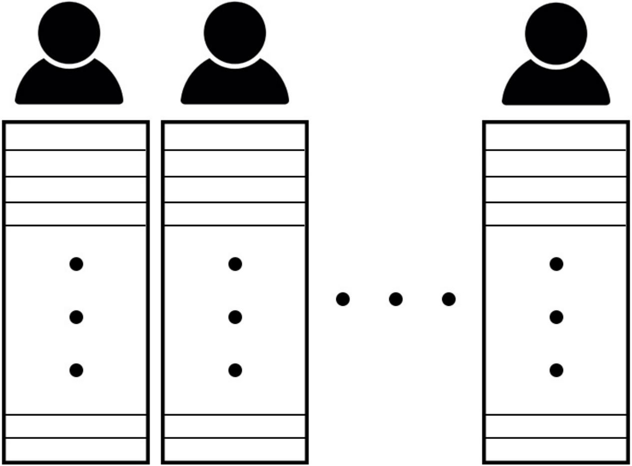
- Compute $\{S^n(X_i), Y_i\}_{i=0}^N$ for some $n \in \mathbb{N}$.
- Apply regression against the truncated signature.

Application to psychiatric data

The problem



Application to psychiatric data

The problem

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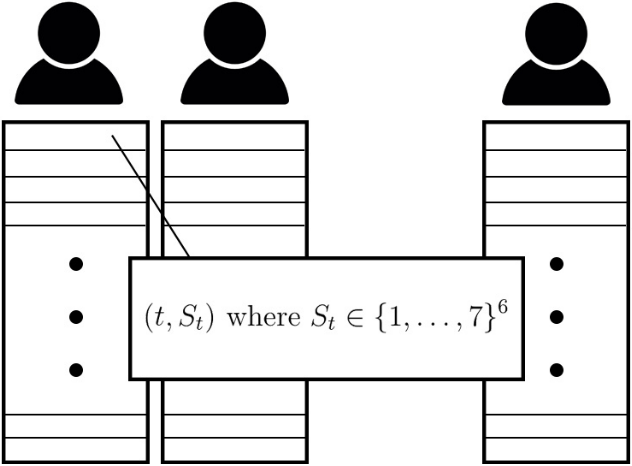


- Given some information about a participant, can we tell if he or she was diagnosed to have bipolar disorder, borderline personality disorder or to be healthy?

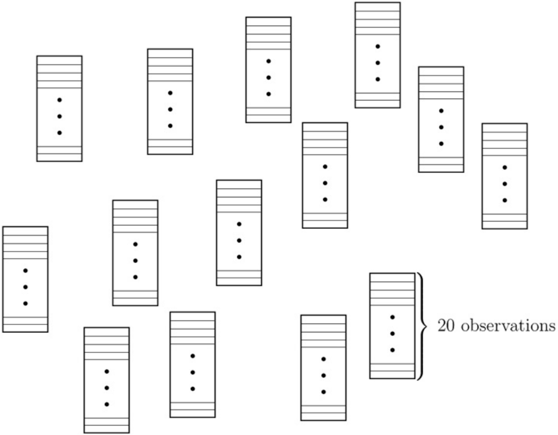
Application to psychiatric data

The problem

- Given some information about a participant, can we tell if he or she was diagnosed to have bipolar disorder, borderline personality disorder or to be healthy?
- Given a participant and information about the last few days, can we predict the mood the following day?

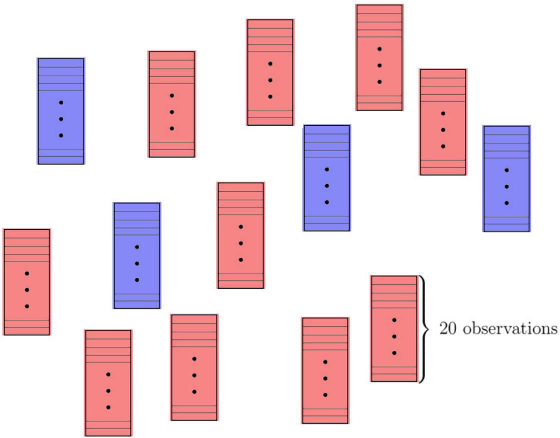
Application to psychiatric data

Methodology



Application to psychiatric data

Methodology



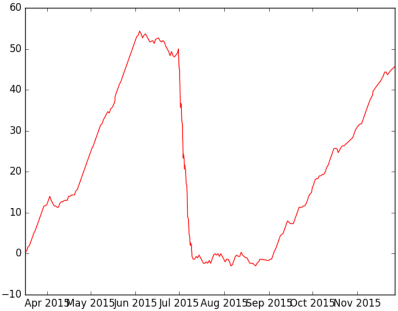
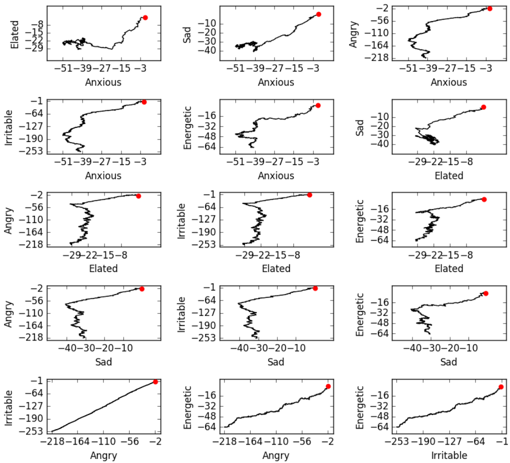


Figure: Normalised path for anxiety scores.


Elated
Anxious
Sad
Anxious
Angry
Anxious
Irritable
Anxious
Energetic
Anxious
Sad
Elated
Angry
Elated
Irritable
Elated
Energetic
Elated
Angry
Sad
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Energetic
Angry
Energetic
Irritable

Application to psychiatric data

Predicting if a person is healthy, has bipolar disorder or has borderline disorder

$$\{t_i, S_{t_i}\}_{i=0}^{19} \quad \rightarrow \quad \begin{cases} (-1,1), & \text{if the partcipant is healthy} \\ (-1,-1), & \text{if the participant is bipolar.} \\ (1,0), & \text{if the participant is borderline.} \end{cases}$$

Application to psychiatric data

Predicting if a person is healthy, has bipolar disorder or has borderline disorder

Order	Correct guesses
2nd	75%
3rd	70%
4th	69%

Table: Percentage of people correctly classified in the three clinical groups.

$$\{t_i, S_{t_i}\}_{i=0}^{19} \quad \rightarrow \quad S \in \{1, \ldots, 7\}^6$$

where S is the scores of the participant the following observation.

Application to psychiatric data

Predicting the future mood

Category	Healthy	Bipolar	Borderline
Anxious	98%	82%	73%
Elated	89%	86%	78%
Sad	93%	84%	70%
Angry	98%	90%	70%
Irritable	97%	84%	70%
Energetic	89%	82%	75%

Table: Percentage of correct guesses for mood predictions

Thank you!