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Using signatures and rough path theory to analyse a psychiatric data stream

Imanol Perez
(Joint work with T. Lyons and K. Saunders)

Mathematical Institute
University of Oxford

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Definition (Signature of a continuous path)

Let $J \subset \mathbb{R}$ be a compact interval, and let $X \in \mathcal{V}^p(J, \mathbb{R}^d)$ so that the following integration makes sense. The signature of X is defined as

$$S(X) = (1, X^1, X^2, \ldots) \in \bigoplus_{n=0}^{\infty} (\mathbb{R}^d)^{\otimes n}$$

where

$$X^n = \int_{\substack{u_1 < u_2 < \ldots < u_n \\ u_i \in J, i = 1, \ldots, n}} \ldots \int dX_{u_1} \otimes \ldots \otimes dX_{u_n} \quad \forall n \geq 1.$$

Definition (Truncated signature of a continuous path)

Similarly, we define, for $n \geq 0$,

$$S^n(X) := (1, X^1, X^2, \ldots, X^n).$$

- We have two data sets: a known set of known input-output pairs (the *training set*), $\{X_i, Y_i\}_i$, which is used to train the model, and a set of inputs that is used for testing (the *out-of-sample set*).

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- Features play an important role in machine learning.

Theorem (B. Hambly, T. Lyons)

The signature of a path with bounded variation is unique up to tree-like equivalence.

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Theorem

Let $X \in \mathcal{V}^1([0,T],\mathbb{R}^d)$ be a path with bounded variation. Then, given a multi-index $I = (i_1,\dots,i_n)$ with components in $\{1,\dots,d\}$ we have

$$\left\| \int_{\substack{u_1<u_2<\ldots<u_n \\ u_i\in J, i=1,\ldots,n}} dX_{u_1}^{i_1} \dots dX_{u_n}^{i_n} \right\| \leq \frac{\|X\|_{1,[0,T]}^n}{n!}.$$

- Given a training set $\{R_i, Y_i\}_{i=0}^N$, of input-output pairs, where $R_i = \{t_{ij}, r_{ij}\}_j$ is a stream of data, construct a new set $\{X_i, Y_i\}_{i=0}^N$ with $X_i \in \mathcal{V}^1$.

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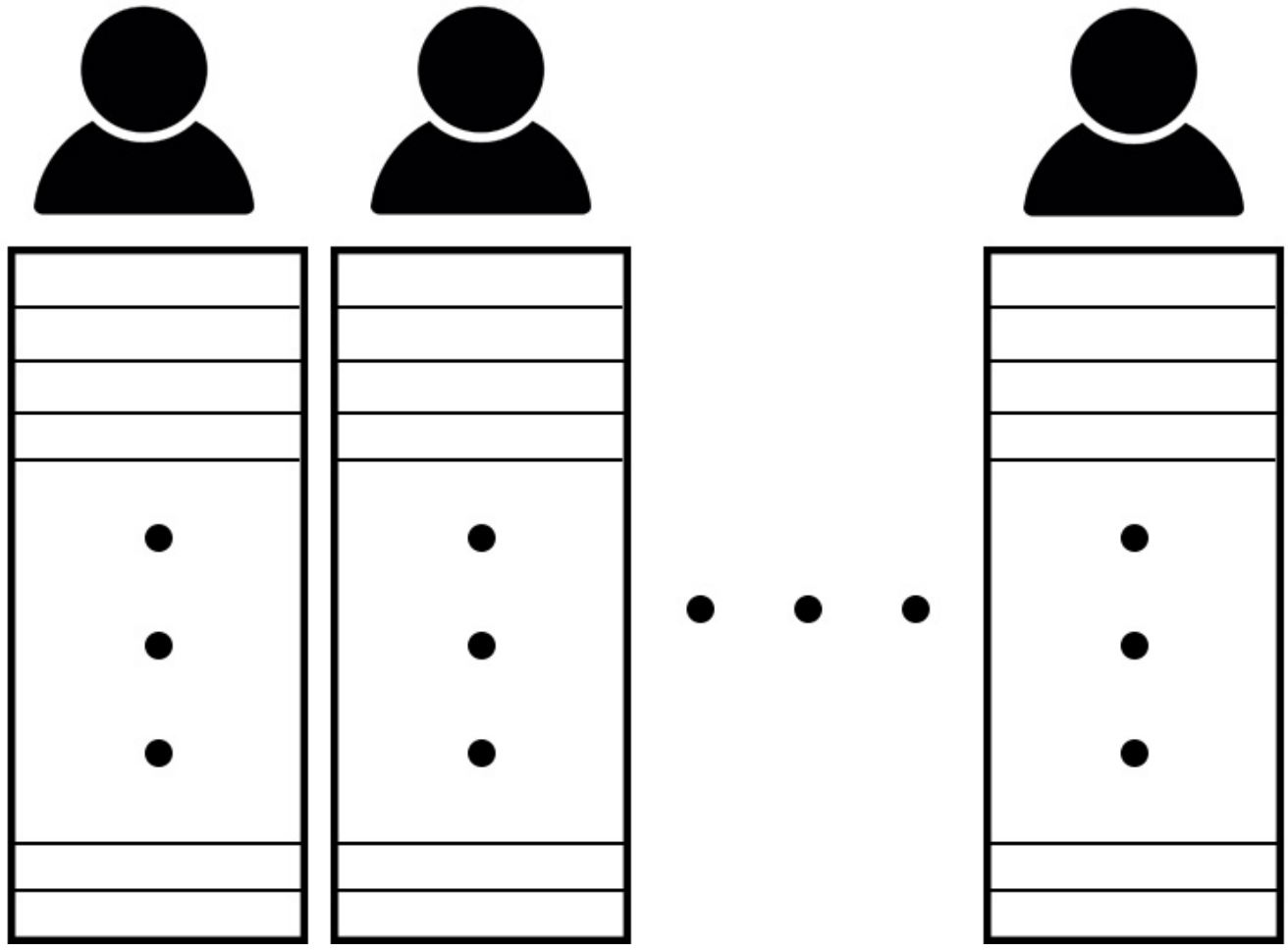
- Apply regression against the truncated signature.

Application to psychiatric data

The problem



Application to psychiatric data

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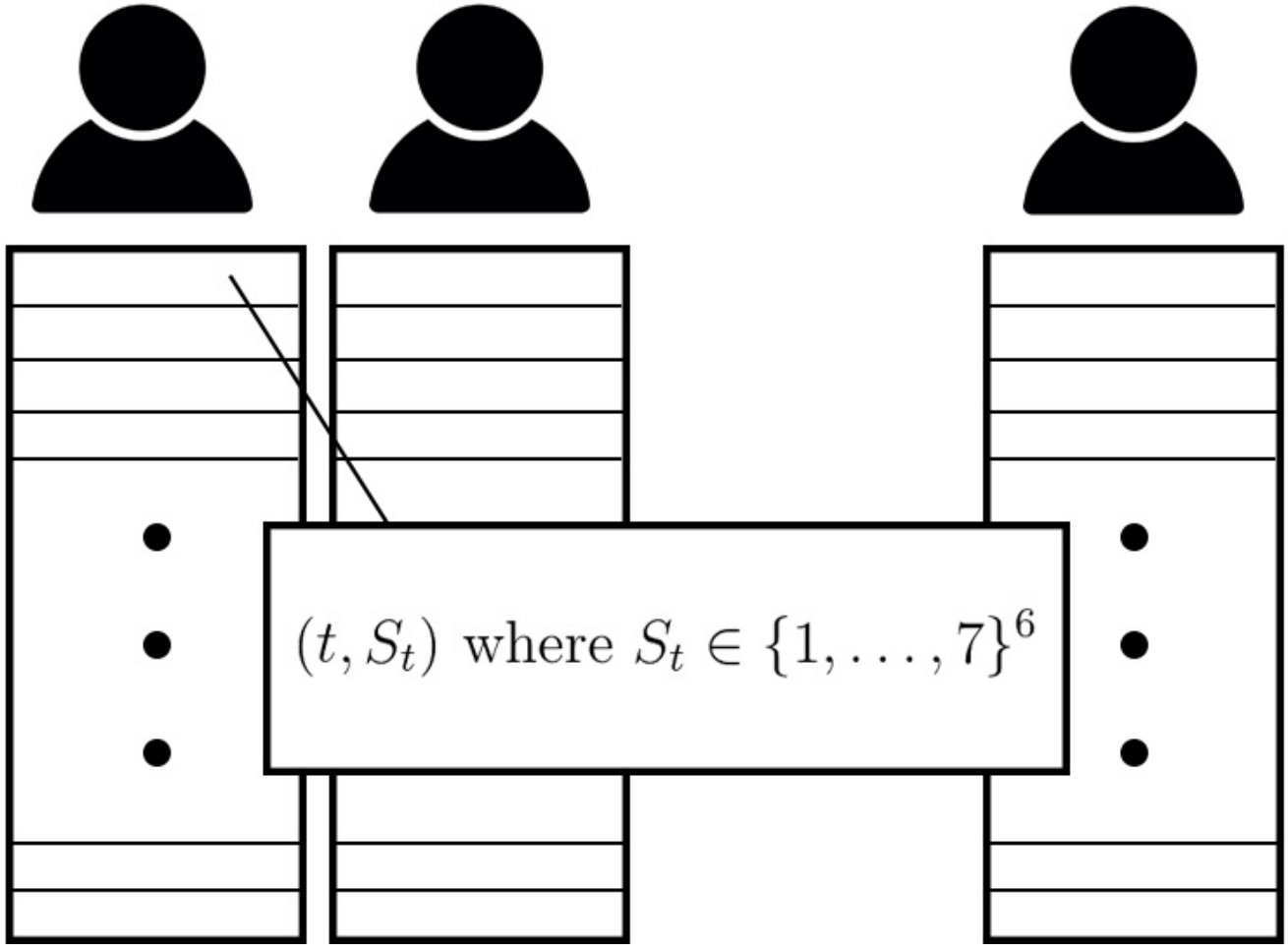

- Given information about a participant, can we tell if he or she is healthy, has bipolar disorder or has borderline personality disorder?

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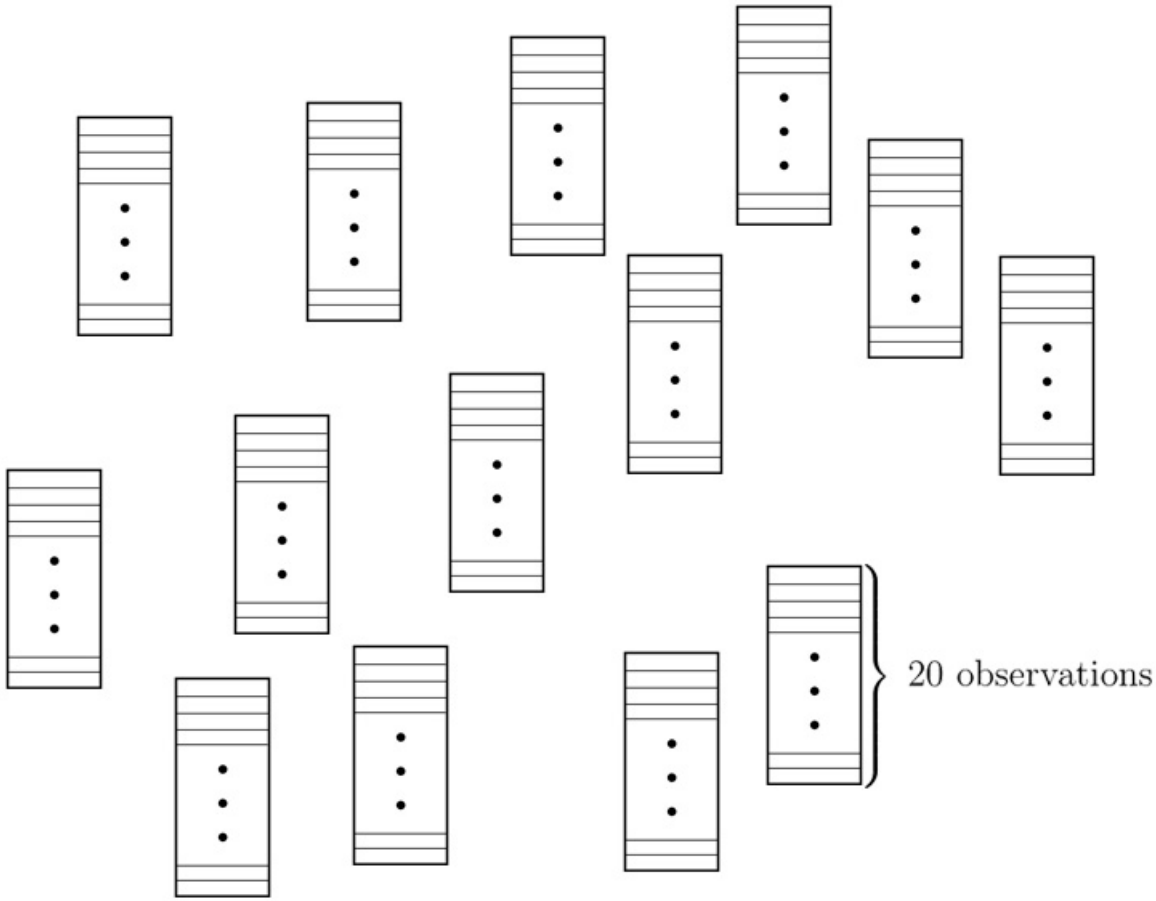
- Given a participant and information about the last few days, can we predict the mood the following day?

Application to psychiatric data

Methodology



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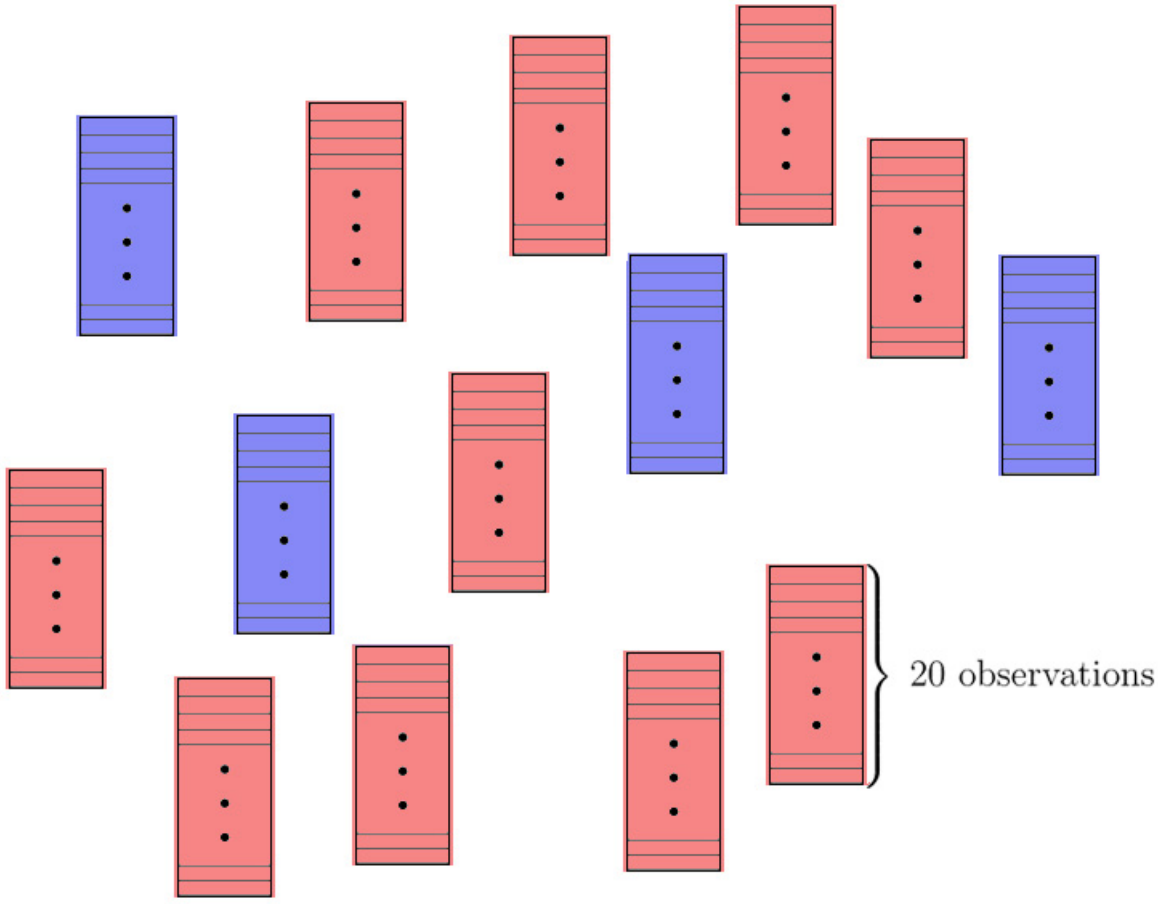

Application to psychiatric data

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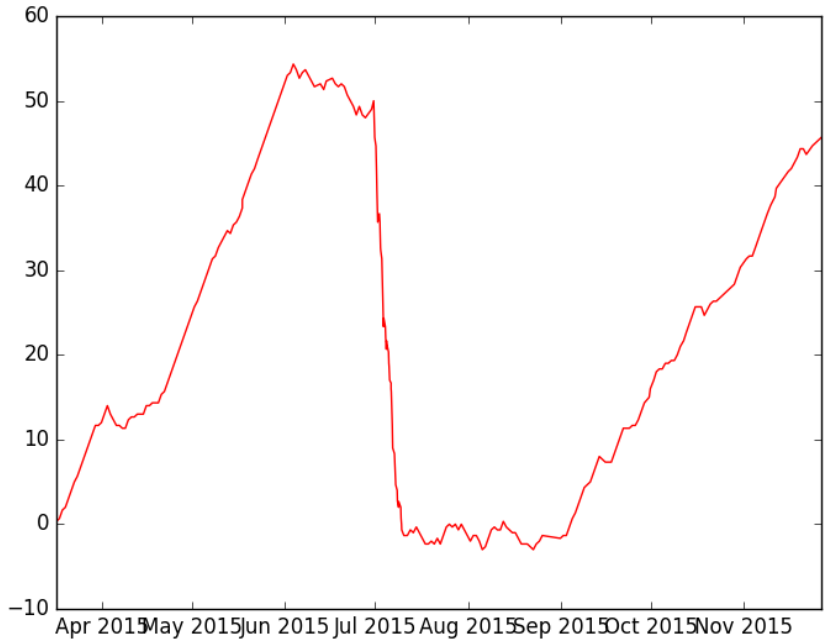

Figure: Normalised path for anxiety scores.

Application to psychiatric data

Predicting if a person is healthy, has bipolar disorder or has borderline disorder

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$$\{t_i, S_{t_i}\}_{i=0}^{N} \quad \rightarrow \quad \begin{cases} (-1, 1), & \text{if the partcipant is healthy} \\ (-1, -1), & \text{if the participant is bipolar.} \\ (1, 0), & \text{if the participant is borderline.} \end{cases}$$

Application to psychiatric data

Predicting if a person is healthy, has bipolar disorder or has borderline disorder

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Order	Correct guesses
2nd	75%
3rd	70%
4th	69%

Table: Percentage of people correctly classified in the three clinical groups.

$$\{t_i, S_{t_i}\}_{i=0}^{N} \quad \rightarrow \quad S \in \{1, \ldots, 7\}^6$$

where S is the scores of the participant the following recorded observation.

Application to psychiatric data

Predicting the future mood of a person with bipolar disorder

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Category	Healthy	Bipolar	Borderline
Anxious	98%	82%	73%
Elated	89%	86%	78%
Sad	93%	84%	70%
Angry	98%	90%	70%
Irritable	97%	84%	70%
Energetic	89%	82%	75%

Table: Percentage of correct guesses for mood predictions

Thank you!

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